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Pricing Insurance and Warranties:
Ambiguity and Correlated Risks

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This paper reports the results of a study of pricing decisions made by professional actuaries. The study formed part of a mail survey of members of the Casualty Actuarial Society conducted to investigate the effects of ambiguity -- in the form of uncertainty about probabilities of risks -- on prices of insurance and warranties. Theoretical hypotheses derived from the expected utility model were compared with the implications of procedures described by practicing acturaries which were found to coincide with several features of the Einhorn-Hogarth ambiguity model. Actuaries were asked to act as consultants to a computer manufacturer concerning the price of a warranty. The design of the study involved comparing ambiguous and non-ambiguous probabilities, different levels of probabilities and size of potential loss, and the nature of risk involved, i.e., whether the risks were indepndent across items or perfectly correlated (if one fails, all fail). Results supported implications of procedures described by actuaris (as well as the Einhorn-Hogarth ambiguity model), but were inconsistent with predictions from expected utility theo

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19. Abstract cont.

regarding correlated risks. In addition to quantitative analyses of the data, further insight is provided by interviews with actuaries concerning their decision-making processes as well as an analysis of comments written on the questionnaire forms. Results are discussed in terms of future work on ambiguity in market settings and the necessary requirements of a model that could provide a more complete account of the data reported here.

Keywords: decision making; economic models

Pricing insurance and warranties: Ambiguity and correlated risks

In 1921, Frank Knight alerted economists to the importance of ambiguity by drawing a distinction between decision making under risk and uncertainty. Today, there is increasing recognition that for many decisions, and particularly those involving low probability events, there is considerable ambiguity associated with the occurrence of specific outcomes. Moreover, although expected utility theory does not recognize Knight's distinction between risk and uncertainty (cf. Howard, 1988), Daniel Ellsberg's (1961) celebrated experimental demonstrations showed that ambiguity can influence decisions in ways that are inconsistent with this important theory.[1]

Many experimental studies have subsequently confirmed Ellsberg's original demonstrations (Becker & Brownson, 1964; Yates & Zukowski,1976; Curley & Yates, 1985) and there has been considerable interest in recent years in incorporating possible effects of ambiguity in both psychological and theoretical models of choice under *uncertainty* (see e.g., Einhorn & Hogarth, 1985; 1986; Hogarth, in press; for the former, and Gärdenfors & Sahlin, 1982; Segal, 1987; Segal & Spivak, 1988; Yaari, 1987; for the latter). Nonetheless, limited empirical data exist on the role that ambiguity plays in the decision-making processes of economic agents such as consumers or professional managers.

This paper contributes to correcting this imbalance by reporting data from a study of decisions made by actuaries related to the pricing of insurance and warranties under conditions involving probabilities that are both ambiguous and known with precision. It thus also continues a line of research by the authors on the effects of ambiguity on insurance decisions (Hogarth & Kunreuther, 1985; in press). Indeed, apart from our own work, we are aware of only one other study that has examined the effects of ambiguity on decisions taken by professionals operating within their area of expertise. This was an unpublished study by Alison Ashton (1984) who found that ambiguity affected the hypothetical decisions made by auditors.

In our earlier work (Hogarth & Kunreuther, in press), we established that ambiguity does affect the pricing decisions of actuaries. Moreover, those specific decisions were consistent with the predictions of the psychological model of decision making under ambiguity proposed by Einhorn and Hogarth (1985;1986). In the present paper, we have sought to deepen our understanding of the decision-making processes of actuaries by (a) interviewing actuaries concerning how they would price risks involving both ambiguous and precise probabilities, and (b) examining the actuarial literature on this topic. Although there is remarkably little actuarial literature concerning ambiguous probabilities, we were able to locate three types of models or procedures that both take account of ambiguity in pricing decisions and are consistent with comments made by actuaries in our interviews.

The models utilized by actuaries suggest that the recommended premiums should be higher when the probability of a specific event occurring is ambiguous rather than non-ambiguous. This rule is not surprising if one places actuaries in the context of their organizational settings where one of their principal roles is to justify specific premium recommendations to underwriters who then make the final decision as to whether or not to insure a given risk. Indeed, Tetlock (1985) has suggested that individuals use an acceptability heuristic in making decisions such that these will be viewed as acceptable to others -- in this case the underwriters. Avoiding ambiguity through higher premiums appears to be one of these rules. Moreover, this behavior is consistent with an explanation recently offered by Curley, Yates and Abrams (1986) who showed in a series of experiments that individuals preferred choices having the smallest degree of ambiguity because these could be justified to others. The higher recommended premiums for ambiguous events may thus provide one explanation as to why there is reluctance by the insurance industry to offer insurance against risks such as environmental pollution liability, political risk, and satellite crashes where probabilities of losses are difficult to assess.

The paper is organized as follows. We first contrast implications of the expected utility model and actuarial procedures for pricing insurance under ambiguity and then briefly describe the Einhorn-Hogarth ambiguity model. Next, differential predictions are tested in an experimental study in which actuaries are asked to imagine that they are advising a computer manufacturer on the price of a warranty. Finally, we discuss our results with respect to effects of ambiguity in market settings and their implications for models of risky choice.

Models for pricing insurance under ambiguity

Risks faced by insurance firms can be classified on many dimensions. In this paper, we examine situations where risks are either independent or perfectly correlated, and where there is or is not ambiguity concerning relevant probabilities.

To illustrate the distinction between correlated and independent risks, imagine insuring homeowners in a particular geographical area. From the viewpoint of the insurer, burglary is an independent risk if a theft in one house has no effect on the probability of thefts occurring in other houses; flood, on the other hand, is an event that either affects all or none of the houses located in a flood plain and thus represents a case of perfect correlation for that risk. The amount of damage may, of course, differ between houses in the same flood plain with the result that correlation will be less than perfect with respect to the magnitude of damage or percentage of the property value affected.

Ambiguity refers to uncertainty concerning the precision of probability estimates and is an important factor when setting prices for insurance (Hogarth & Kunreuther, in press). Imagine, for instance, being asked to quote premiums with respect to launching a satellite from an orbiting space vehicle[2] or insuring a businessman against risks of kidnap or hijacking during a visit to the Middle East. How does one estimate the chances of such events occurring given limited past data on which to base a judgment?

For practical purposes, it is important to specify the source of ambiguity since this affects how one chooses to model it. In this paper, ambiguity reflects the fact that several experts have provided the decision maker with different probability estimates.[3] Thus, some aggregation procedure is required (see e.g., Genest & Zidek, 1986). We shall examine the effects of ambiguity by computing the weighted averages of k different expert opinions. Specifically, imagine situations involving j (= 0,...,m) possible losses and let p_{ij} represent the estimate by expert i (=1,..,k) of the probability that j losses could occur. Assume that each expert's estimate is accorded a weight w_i such that $p_{.j} = \sum_{i=1}^{k} w_i p_{ij}$ and $\sum_{i=1}^{k} w_i = 1$. We will compare this situation with the mathematically equivalent case of an unambiguous probability estimate $p_j = p_{.j}$.

The expected utility model. In economics, this model provides the basic analytic framework for considering insurance decisions (see e.g., Ehrlich & Becker, 1972). One of the main predictions made by this model is that, under competitive conditions, insurers will set prices (i.e., premiums) for a given type of loss at a level that leaves them just *indifferent* between insuring and not insuring the risk. More formally, consider an insurer who has no ambiguity concerning relevant probabilities and is considering insuring m independent, identical risks each of which could result in either a loss of L with probability p or zero with probability (1-p). According to expected utility theory, the insurer will set the same premium for each risk at a level such that he or she is indifferent between insuring all of the m risks or not insuring any of them. This is given by

$$U(W) = \sum_{j=0}^{m} p_j \, U(W - jL + mr_1) \qquad (1)$$

where W represents the firm's wealth, p_j is the probability of incurring j of m possible losses (as defined previously), U(.) is the firm's utility function, and r_1 is the premium where the insurer is indifferent between offering and not offering coverage.

Define r_2 as the premium for independent risks with ambiguous probability estimates. By incorporating the estimates of the different experts, Equation 1 can be modified for the case of independent risks and ambiguous probabilities to the form

$$U(W) = \sum_{i=1}^{k} w_i \left[\sum_{j=0}^{m} p_{ij}\, U(W - jL + mr_2)\right] \tag{2}$$

By comparing Equations 1 and 2, note that the expected utility model predicts no difference between premiums in the ambiguous and non-ambiguous conditions if one assumes that insurers' utility functions are linear (i.e., with a linear utility function, the expectations of the right hand side of Equations 1 and 2 are equal). However, although economists have typically assumed that insurers are risk-neutral, to the best of our knowledge there is no empirical evidence supporting this assumption.[4] Instead, we believe that insurers are more likely to be risk-averse and show in Appendix A that, for insurers with utility functions exhibiting risk aversion, premiums will be higher when based on ambiguous probabilities. In other words, the expected utility model predicts $r_2 > r_1$.[5]

For perfectly correlated risks, there can be only either 0 or m losses. If r_3 is defined as the indifference premium for perfectly correlated risks with non-ambiguous probabilities, Equation 1 can be simplified to

$$U(W) = (1 - p_m)\, U(W + mr_3) + p_m\, U(W - m[L - r_3]) \tag{3}$$

For *ambiguous probabilities*, let r_4 be the analogous indifference premium and the resulting equation is

$$U(W) = \sum_{i=1}^{k} w_i \left[(1 - p_{im})\, U(W + r_4) + p_{im}\, U(W - mL + mr_4)\right] \tag{4a}$$

Since $p_m = \sum_{i=1}^{k} w_i p_{im}$, this can be rewritten as

$$U(W) = (1 - p_m)\,U(W + mr_4) + p_m\,U(W - m[L - r_4]) \qquad (4b)$$

Comparing Equations 3 and 4b, note that $r_3 = r_4$ thus implying that the expected utility model predicts *no differences in premiums* due to ambiguity when losses are perfectly correlated no matter what the shape of the utility function. Although this result may seem counterintuitive, recall that the case of perfect correlation under ambiguity is equivalent to insuring a single risk with a specific probability where ambiguity does not affect premium levels if the insurer maximizes expected utility (Hogarth & Kunreuther, in press).

Assuming a risk-averse insurer, the expected utility model also makes a prediction concerning the difference between premiums for perfectly correlated and independent risks. This is that the indifference premium for correlated risks will be higher than the indifference premium for independent risks since the former implies a greater probability of a large loss. Thus $r_3 > r_1$ and $r_4 > r_2$. To summarize, the expected utility model makes the following predictions concerning the relative sizes of the four cases considered above: $r_4 = r_3 > r_2 > r_1$.

It is also instructive to consider the expected utility predictions for changes in the size of the potential loss as operationalized here by the number of risks underwritten, i.e., m. Increasing the number of risks induces the counteracting effects of increasing the potential total loss but decreasing uncertainty (i.e., variance) concerning the magnitude of the loss as risks are less correlated. The net effect of these contrary forces on the indifference premium depends on the specific utility function. Risk-averse insurers will want to raise indifference premiums as m increases. In addition, the more highly correlated the losses, the more indifference premiums will be raised.

Actuarial procedures. One goal of the present work is to understand how professionals (i.e., actuaries) handle pricing decisions under conditions of ambiguity. Although we initially sought illumination in the extensive literature actuaries are required to know for their professional examinations, we were disappointed. Actuarial science is remarkably well developed for cases where probabilities are known with precision, but relatively little formal literature exists to guide actuaries in how to handle ambiguous probabilistic information. Thus, to gain insight into the process by which actuaries make decisions under conditions of ambiguity, we conducted an informal focus group with several actuaries from a large insurance company. Our discussions revealed that pricing under ambiguity depends largely on making subjective adjustments to formulae that would normally be applied to precise probability estimates. Moreover, such adjustments are judgmental in nature.

Specifically, the pricing of insurance is characterized by a two-stage process.[6] First, since the essence of insurance depends on the law of large numbers, insurers seek to establish portfolios involving large numbers of independent risks. Thus, any new risk is initially evaluated in terms of its maximum possible loss and how its addition would affect the insurer's total portfolio of risks. In other words, new risks are evaluated within the context of a firm's existing portfolio of risks. Second, conditional on being deemed acceptable, the price reflects both estimates of the probabilities of incurring losses and ambiguity or uncertainty concerning such estimates. To a large extent, the procedures advocated are consistent with taking decisions based on expected utility theory with a risk-averse utility function; however, the actual premiums recommended by actuaries will vary depending on the adjustment procedures utilized. We now consider three different types of procedure.

Procedure 1 -- subjective ambiguity adjustment. This procedure explicitly recognizes the effects of ambiguity by inflating premiums based on expected value by a factor reflecting the amount of perceived ambiguity as well as random fluctuations. In this sense, it is viewed by actuaries as a global security loading (Lemaire, 1986). Thus, denoting a premium calculated on the basis of expected value by μ, the premium charged is given by the formula

$$r = (1 + \alpha)\,\mu \tag{5}$$

where α ($\alpha > 0$) is the factor reflecting ambiguity (although there is no discussion in the literature as to how α should be determined).

Procedure 2 -- mean-variance models. The second class of procedures recognizes the existence of variance due both to inherent variability in outcomes and the presence of ambiguity. These suggest adjusting estimates of premiums based on expected value by an amount that is a function of the estimated variance (Lemaire, 1986). Thus,

$$r = \mu + \lambda f(\sigma) \tag{6}$$

where $f(\sigma)$ is either the variance or standard deviation of the estimated distribution and λ ($\lambda > 0$) is a constant that reflects the extent to which $f(\sigma)$ should influence r. (Once again, however, little guidance is offered in how to set λ).

Procedure 3 -- Constrain risk of ruin. The third procedure may be used in conjunction with others but does not address the impact of ambiguity directly. Simply stated, it suggests setting premiums at a level such that the probability of depleting the insurer's reserves is held below a certain level. More formally, it can be stated by the rule: Select the minimum r that satisfies the condition that

$$P\{|m(L - r)| > R\} < \gamma \tag{7}$$

where R represents a specific amount of reserves, ($R \leq W$), and γ has been defined according to the insurer's policy. This approach has been discussed in some detail by Stone (1973) who indicated that insurers are interested in maximizing expected profits within certain constraints of tolerable risk and stability of operations. Specifically, insurers want to limit the risk of insolvency. One way to do this is to set premiums high enough based on a given number of policies so that the probability that their losses plus expenses exceed their income is below a prespecified level.[7] The value of γ reflects the insurer's concern for safety and will determine the rates which will be set for a given sized portfolio of risks. According to this procedure, the more highly correlated are the risks, the larger the premium.

The use of Equation 7 by actuaries is very much in the spirit of a safety-first model which postulates a threshold level of probability related to some target level of performance -- in this case adequate reserves. In discussing managerial perspectives on risk taking, March and Shapira (1987) have emphasized the importance of such focal values particularly in situations where the probability of loss is poorly specified or ambiguous.

These different procedures leave much to the judgment of actuaries with practice varying between different insurers or types of insurance coverage. Taken together, the procedures are consistent with the following predictions:

(a) Prices will be higher when probabilities are ambiguous as opposed to non-ambiguous (Procedures 1 and 2), i.e., $r_2 > r_1$ and $r_4 > r_3$.

(b) Prices will be higher when risks are perfectly correlated as opposed to being independent (Procedures 2 and 3), i.e., $r_3 > r_1$ and $r_4 > r_2$.

(c) Prices per dollar of insurance coverage will be higher when the size of the potential loss increases (Procedure 3).

These predictions are summarized in Table 1 together with the corresponding predictions of the expected utility model.

Insert Table 1 about here

A psychological model. As noted above, the procedures suggested in the actuarial literature leave much scope for subjective judgment. It is therefore of interest to note how the implications of these procedures compare with those of the psychological model developed by Einhorn and Hogarth (1985; 1986) for describing how people assess and weight ambiguous probabilities in decision making. The model, which is described more fully in Einhorn and Hogarth (1985; 1986) and Hogarth (in press), is based on an anchoring-and-adjustment process whereby people are first assumed to anchor estimates of ambiguous probabilities on a specific figure, and then to adjust this by imagining, via a mental simulation process, alternative values the probability could take.

For example, consider a situation in which an insurer is concerned about the chances of an accident occurring in a factory. Although technical experts assess the risk as $p = .001$, there are doubts about the precision of this estimate. In the Einhorn-Hogarth model, it is assumed that people would first anchor on a given value of probability (e.g., the .001 provided by the experts) and then imagine or "try out" other values the probability could take, both below and above the anchor. Depending on the circumstances, equal weight would not necessarily be accorded in imagination to possible values of the probabilities on both sides of the anchor. For instance, in the present example values above the anchor may well weigh more heavily in imagination than those below because of the more negative consequences associated with a higher rather than lower probability of loss.

In the Einhorn-Hogarth model, the amount of the mental simulation reflects the amount of perceived ambiguity (the greater the ambiguity, the greater the simulation), whereas the net effect of weight accorded to values below and above the anchor reflects (a) the location of the anchor (for low values of the anchor, the range of possible values above the anchor is greater than that below, and vice versa), and (b) the relative importance attached in imagination to possible values of the probability above as opposed to below the anchor. Thus, assuming that the sign and size of payoffs influence the imagination of prudent decision makers, estimates of ambiguous probabilities of losses will tend to exceed their corresponding anchor values, while for probabilities of gains it is the reverse.[8] Similarly, the larger the payoff, the greater the net effect of the over- or under-adjustment relative to the anchor value.

For low probability of loss events, therefore, the Einhorn-Hogarth model implies larger premiums for ambiguous as opposed to non-ambiguous situations, for correlated as opposed to independent risks (since the potential loss is greater for the former), and for ambiguous risks involving large as opposed to small payoffs. As such, the predictions of the Einhorn-Hogarth model mimic those of the actuarial procedures.

Experimental evidence

Rationale. Table 1 contrasts the predictions of the actuarial procedures with those of the expected utility model. All the models predict that premiums will be higher under ambiguity if risks are independent. When risks are perfectly correlated, the actuarial procedures predict that ambiguity will lead to higher premiums than if the probabilities were known with certainty. For this case, however, expected utility theory predicts that ambiguity will have no impact on premiums. Finally, the effect of increasing the size of the potential loss will be to increase the premium charged when actuarial procedures are utilized. A similar prediction would be made for the expected utility model when risks are correlated; for independent risks, the effect on premiums charged depends on the shape of the insurer's utility function. A principal goal of the study is to test these predictions.

Subjects. The subjects were professional actuaries who responded to a mail survey of members of the Casualty Actuarial Society residing in North America in January 1986. Of this population, 489 of 1,165 persons (i.e., 42%) provided usable responses. Mean length of experience as actuaries reported by the respondents was 13.8 years (median 12 years) with a range from 1 to 50 years. Responses were provided anonymously. The actuarial profession is one of the smallest (in total membership), highest paid, extensively trained, and specialized in North America. Analyses made by actuaries are key inputs to pricing decisions made by insurance companies.

Survey instruments and design. Packages containing the survey instruments were mailed to the actuaries with stamped addressed envelopes provided to facilitate returns. Each package contained (a) a letter from one of the authors requesting participation in a study on risky decision making, (b) a letter from the Vice-President for Development of the Casualty Actuarial Society also urging participation, and (c) the survey questionnaire. This consisted of two or three scenarios, each of which appeared on different sheets of paper that had been stapled together in booklet form.

Spaces for responses were provided and indicated on the sheets. Respondents were told that they would "find a number of questions related to the pricing of insurance and warranties in different scenarios." They were requested to answer the questions in the order in which they appeared.[9] Respondents were also asked to indicate their "length of experience as an actuary, in number of years."

The scenarios had been pretested prior to use in the survey with two groups of actuaries in Chicago as well as by officials of the Casualty Actuarial Society. In designing the scenarios, we made an explicit trade off between providing the respondents with the kind of detailed background information that would normally accompany real actuarial cases and making the scenarios short enough so that members of a busy profession would not be discouraged from responding. The survey required approximately 15 minutes to complete since each respondent's task was limited to at most three short scenarios. Care was exercised to constitute combinations of conditions and scenarios that would minimize possible "carry-over" effects. In addition, the order of scenarios within combinations was randomized. Finally, members of the respondent population were also randomly assigned to different combinations of scenarios and conditions.

To determine whether ambiguity does or does not affect behavior, it is important to ensure that subjects perceive the size of probabilities in both the ambiguous and non-ambiguous cases to be equal such that the stimuli differ only in respect of ambiguity (see, e.g., Ellsberg, 1961; Hogarth & Kunreuther, in press). To achieve this requirement, we adopted the procedure of telling respondents in the ambiguous conditions that a particular number (e.g., .20) was "*your* best estimate" (emphasis added here) even though it was based on uncertain information (i.e., experts disagreed) and they could not be too sure of it. The same value was given to subjects in the corresponding non-ambiguous case except that they were told that the probability could "be confidently estimated at ..." Respondents in the ambiguous conditions were therefore endowed with the belief that the probability on which they should anchor their initial judgment was the same as that manipulated in the corres, onding non-ambiguous situation.

Scenario. Respondents were asked to assume the role of an actuary called in by Computeez, a manufacturer of personal computers, to determine the price of a one-year warranty on the performance of a new line of microcomputers to be put on the market during the coming year. The warranty was to cover the failure of the XY component manufactured by Computeez. The cost of repairing a breakdown was stated to be $100 per unit. Respondents were also informed that there could be at most one breakdown per unit during the warranty period.[10]

Experimental variations concerned (a) two levels of the number of units that Computeez expected to sell, viz., 10,000 and 100,000, (b) ambiguous versus non-ambiguous probabilities of breakdowns, (c) whether the risks of breakdowns associated with any computer were independent of other computers sold or would be common to all computers (i.e., the insured risks could either be independent across individual units or perfectly correlated), and (d) three different probability levels concerning the risk of XY component failure; these were: .001, .01, and .10.

Since disagreement among expert opinions was identified as the source of ambiguity in our theoretical analysis of predictions of the expected utility model (above), this was explicitly incorporated in the construction of the ambiguous versions of the scenario. Respondents were told that experts were confused by the results of tests concerning the performance of the XY component, that there was considerable disagreement amongst the experts concerning the chances of a breakdown, and that respondents should not be "at all confident in the accuracy" of their estimate of the probability of a breakdown. On the other hand, in the non-ambiguous versions of the scenario experts had examined company records, conducted several independent tests of their own, and all agreed on the chances of the XY component becoming defective within a year of purchase such that the probability of this event could be confidently estimated.

Independence of the probability of breakdown of the XY component in different computers was noted by stating that the nature of the potential flaw was random rather than systematic across computers. Dependence was indicated by stating that the potential flaw was due to a particular aspect of the manufacturing process so that if the XY component failed in any one E-Z computer, it would fail in all others as well.

Respondents were asked to state prices on a *per unit* basis, specifically: "What is the minimum pure premium you would recommend for the warranty (per unit sold) on the understanding that this will cover the $100 per unit cost of repairing the XY component if this fails within a year of purchase?"

Design. The study involved four between-subject factors. These were: size of potential loss with 2 levels at 10,000 and 100,000 units; ambiguous and non-ambiguous probabilities; independent versus correlated risks; and three levels of probability of a breakdown at .001, .01, and .10. There were thus 24 different between-subject conditions, i.e., $2^3 \times 3$.

Results. Of the 489 actuaries who responded to the survey (i.e., 42% of the population), 468 provided usable responses for this study, six stated that they would refuse to insure (see below) and 15 failed to respond to this questionnaire although they did answer other questions in the survey.

The main results of the experiment are presented in Table 2 in the form of mean and median prices for all 24 experimental conditions. A note at the foot of the table indicates all those variables and interactions that were found to be statistically significant ($p < .10$) by an appropriate analysis of variance.

Insert Table 2 and Figures 1 and 2 about here

Results concerning the first three predictions detailed in Table 1 are summarized graphically in Figure 1. This shows that the statistically significant main effects for ambiguity and type of risk (see Table 2) have distinct additive effects on stated premiums. The ordering of the different types of premium is $r_4 > r_3 > r_2 > r_1$. This contradicts the prediction of expected utility theory that $r_4 = r_3$, but is consistent with the actuarial procedures (and the Einhorn-Hogarth model).

Further evidence supporting the effects of both correlated risks and ambiguity comes from examining the types of situation faced by the six respondents who replied that they would refuse to insure. Five of these six faced ambiguous, correlated risks. (The sixth person was in the ambiguous/ independent cell of the research design).

With respect to Prediction 4 (Table 1), Table 2 shows that there was a main effect for number of units insured which was significant at $p < .10$; the different mean prices per unit were \$7.09 for 10,000 units and \$8.48 for 100,000 units. In addition, there is a two-way interaction ($p = .07$) between ambiguity and number of units at risk which is presented graphically in Figure 2. This shows that, whereas in the absence of ambiguity there was no difference in mean price per unit between the 10,000 and 100,000 unit conditions (i.e. \$6.48 and \$6.37, respectively), there was a considerable increase under ambiguity from a mean price per unit of \$7.69 for 10,000 units to \$10.59 for 100,000 units.

Changes to the probability of failure produced a significant and anticipated main effect -- see Table 2.[11] The average premiums per unit loss across the three probability levels of .001, .010 and .100 were, respectively, \$1.94, \$5.06, and \$16.35; moreover, each of these are considerably larger than premiums based solely on expected value (see top of Table 2). However, the increases in premiums are disproportionally smaller than the increases in probability levels. To see this, note that if the average premiums at each probability level were proportional to increases in probability, they would be \$1.94 at .001, \$19.40 at .010, and \$194.00 at .100!

Qualitative analysis. Whereas it is clear that the actuaries' decisions do not adhere to all implications of the expected utility model, it is difficult to say which combinations of the different actuarial procedures they might have followed and/or the extent to which the Einhorn-Hogarth model adequately represents their decision-making processes. Fortunately, some light can be shed on the decision processes actually followed by some of the actuaries because several spontaneously made notes on their questionnaires of the line of reasoning followed in arriving at their responses. To exploit these data, the following procedures were followed.

First, the 489 usable questionnaires received from the survey were classified into two groups:

those which only contained responses to the questions posed, and those where the actuaries had also written comments, e.g., steps involved in calculations. This criterion was applied to all notes on the questionnaires and was not limited to the questions concerning the study presented in this paper (see also Hogarth & Kunreuther, in press). The comments of 70 of these 89 respondents referred specifically to the reasoning underlying their responses and could be sorted into four different groups. These were:

	Number of responses
1. Explicit mention or calculation of expected value but then answering by giving a different (higher) price without any other indication of how the latter was determined	30
2. Explicit mention or calculation of expected value which was then the answer given	20
3. Calculation of expected value but with an adjustment factor to account explicitly for the "risk" involved. This corresponds to the actuaries' Procedure 1 -- subjective ambiguity adjustment.	15
4. Explicit use of the actuaries' Procedure 2 -- mean-variance models	5
	70

The most striking feature of these data is the important role played by expected value, either as the method for establishing the price (the second group of responses) or as an intermediate step in reaching a final price (the three other groups). In other words, these data support the hypothesis that expected value forms an "anchor" which the actuaries adjust in determining a price. Moreover, this judgmental strategy was explicitly and independently described by actuaries in our focus-group interviews. As such, this process is consistent both with the 1st and 2nd actuarial procedures but disconfirms a process detail of the Einhorn-Hogarth model since in the latter the anchoring-and-adjustment is postulated to apply only to the probabilistic component of the price.

Since for 30 of the 70 responses (i.e., the 1st group) no indication was provided as to how the actuaries adjusted expected value in determining prices, it is instructive to examine some of the detailed responses in the 3rd and 4th groups that correspond to the 1st and 2nd actuarial procedures.

Consider the following line of reasoning given by one actuary in quoting a premium of $.12 for an ambiguous .001 probability of incurring a $100 loss: " 100 x .001 = $.1 (want 20% hedge)."

Another actuary illustrated the same general method when asked to quote for a potential $100,000 loss at two levels of ambiguous probabilities, .01 and .65.[12] For .01, the line of reasoning was given as:

"(.01) (100,000) = 1,000 / x (100/70)/ 1,429 => 1,450"

In other words, the actuary first calculated expected value (i.e., 1,000), and then adjusted this by a factor of 100/70 to yield 1,429 which was rounded up to 1,450.

For .65, the reasoning provided was:

"(.65) (100,000) = 65,000 / x(100/80) => 81,250"

The prices quoted for the .01 and .65 probabilities were $1,450 and $81,250 respectively. Not only does this example follow the ambiguity adjustment method, but it is of particular interest to note that the actuary uses different coefficients to adjust expected value at the two probability levels, with the smaller probability receiving the larger adjustment. In fact, this treatment is consistent with an implication of the Einhorn-Hogarth model that the net adjustment to the anchor should decrease with the size of the probability of loss (see Hogarth & Kunreuther, in press).

A further example of the ambiguity adjustment method was provided by an actuary who had been asked to quote for a .35 probability of a $100,000 loss. In one scenario given to this actuary, the probability was ambiguous; in another, it was non-ambiguous. The notes attached to the responses for the ambiguous and non-ambiguous cases were, respectively,

"100 x .35 x 1.25
↑ = 43,750
Conf factor"

and

"100,000 x .35 x 1.0"

with a response of $35,000. In other words, the same scenario induced a 25% upward adjustment in premium when the probability was ambiguous as opposed to none in the non-ambiguous case. (We assume that by "conf factor" the actuary was referring to a subjective "confidence factor" based on an assessment of the experimental materials).

There were only five examples of explicit mean-variance kinds of calculations. The gist of the arguments given in these cases was to write the premium as the sum of expected value plus a coefficient (λ) multiplied by an estimate of the standard deviation of the probable loss. In a couple of cases, comments were made as to values chosen for λ, e.g., "$\lambda = 1$," "$\lambda = 0$," or "λ = degree of risk. Select 10%."

Of the 19 persons whose comments could not be allocated to one of the four categories provided above, it is significant to note that several complained that the scenarios failed to provide information concerning the amount of loss the insurance companies or manufacturers could afford to risk in the different scenarios. It was therefore not surprising that no explicit calculations appeared to follow the 3rd actuarial procedure ("Constrain risk of ruin"). However, the presence of these comments clearly indicates that the amount of reserves companies can put at risk in any particular line of business is an important consideration in setting premiums as suggested by the insightful analysis of Stone (1973). It also reinforces the existence of the two-stage process of pricing described above (i.e., first decide if the risk is acceptable given the context of one's portfolio of risks, then set the price if it is acceptable).

To obtain further insight into the decision processes of actuaries, we obtained the cooperation of five actuaries who agreed both to respond to some scenarios and provide specific comments as to how they determined prices.[13] Analyses of these responses showed that the actuaries were particularly sensitive to both ambiguity and the difference between correlated and independent risks. For example, in commenting on a Computeez scenario involving a correlated, ambiguous risk where the best estimate of the probability of a breakdown was stated to be .10, one actuary specifically enumerated the following points: (1) perfect correlation between risks implies no spread and therefore greater exposure; (2) lack of confidence in the probability estimate greatly increases the risk; (3) adding a risk of this type to the insurer's portfolio of risk greatly increases the variance of risks faced; and (4) concerns about risk of ruin, and a "substantial hit to earnings" should a loss be incurred with the latter implying all kinds of ramifications involving the company's directors, stockholders, regulators, the trade press, and so on. He finished by stating that he would either be inclined to refuse to insure the risk or, at least, demand a premium that was "near 100 cents to the dollar."

Another actuary indicated that actuaries approach the rate-setting process from two perspectives: (a) utilizing expected value to make the best decision but (b) having an obligation to set prices above expected value to keep the company financially sound and prevent insolvency. This line of reasoning supports the use of either procedures 1 or 2 described above.

General discussion

As summarized in Figure 1, this study has documented large effects of ambiguity and correlated risks on prices. Contrary to the predictions of expected utility theory, actuaries' pricing decisions are sensitive to the presence of ambiguity when risks are correlated. This paper therefore raises important issues concerning the effects of ambiguity in market settings and the types of decision-making models needed to account for the experimental results.

Ambiguity about probabilities is a common phenomenon. In addition to the obvious examples of insurance and warranties discussed here, consider decisions such as selling or buying new products, attempts to introduce social or technical innovations, medical testing and the use of certain procedures on patients, the risks surrounding new technologies, and the appointment of key personnel in organizations. In all of these cases there is considerable ambiguity associated with the probability of success or failure. Indeed, it is surprising that ambiguity has not been the subject of more empirical work outside experimental laboratories.

We suggest three reasons why this is the case. First, many economists limit their attention to equilibrium behavior in markets that by definition involve repetitive decisions taken by "professionals," the stock market being a prototypical example. These studies provide precisely the kind of data in which one would not expect to see important ambiguity effects in that buyers and sellers receive quick feedback from which they can learn (for experimental evidence, see Camerer & Kunreuther, 1988). On the other hand, within these kinds of market it would be interesting to examine decision making over time to assess whether ambiguity impacts on inexperienced agents at early stages of the process (cf. Arrow, 1982). Second, the quality and aggregate nature of economic data make it difficult to distinguish between "distortions" in probability due to ambiguity and genuine differences in beliefs about underlying probabilities. Thus, to the extent that trading in markets reflects differences in beliefs (Varian, 1986), possible asymmetries in the effects of ambiguity on different parties to economic transactions take on added importance. Third, the wide-scale adoption of the expected utility paradigm in applied economics avoids the issue of ambiguity. Only recently have formal models been developed that are capable of predicting ambiguity effects (see Fishburn, 1986; 1988; Kahn & Sarin, in press; Segal, 1987; Segal & Spivak, 1988; Yaari, 1987).

It would be illuminating to study the effects of ambiguity in different market settings. For example, one class of decisions examined by Hogarth (in press) concerns situations where the two sides to a transaction (e.g, buyers and sellers, plaintiffs and defendants) are affected differentially by the impact of ambiguity. This arises because of a framing effect (cf. Tversky & Kahneman, 1981) where one party naturally encodes a situation in terms of a probabilistic loss and the other as a probabilistic gain. For certain ranges of probabilities of events, the Einhorn-Hogarth model implies that whereas the decision of one party to the transaction will be sensitive to ambiguity, the decision of the other will not. This, in turn, can imply strategic advantages and disadvantages in how the parties negotiate the transaction. It would be important to investigate whether the experimental validation of these predictions obtained by Hogarth (in press) would be replicated under market conditions such as buying and selling options in futures markets.

Another relevant class of situations concerns asymmetries in information between two parties to a transaction. Consider, for example, differences between ambiguous buyers (e.g., consumers or industrial firms) and non-ambiguous insurers. According to the Einhorn-Hogarth ambiguity model, buyers should often be prepared to pay much more for coverage than insurers will necessarily want to charge. We would therefore expect to find active insurance markets for such risks. Some that come to mind are automobile insurance, warranties for consumer durables, and insurance against risks that are relatively rare such as airplane accidents (Eisner & Strotz, 1961) or certain forms of illness. To the student of economic markets, therefore, an interesting issue centers on the extent to which competition forces insurers to reduce prices so that any excess rents due to ambiguity are eliminated.

Ambiguity may also contribute to the failure of insurance markets since basic conditions of insurability are not met.[14] For example, in a recent survey of the insurance industry for *The Economist*, McCullough (1987) stated,

> Some of the areas which insurers refused to cover in 1985-86 were: pollution risks, liquor liability (that is, cases related to drunken driving), day-care centres, medical malpractice, asbestos removal from schools, commercial fishing boats, municipal liability, commercial trucking and high-limit coverage (above $50m) for industrial concerns. The insurers argued that they were pulling out because liberal court decisions made the potential losses incalculably large, or because the risk itself was no longer actuarially predictable.

Note that McCullough mentions two sources of ambiguity. One is the estimation of risk that has been discussed in this paper; the other is ambiguity concerning the amount of the potential loss covered by insurance (see also Priest, 1987). The failure of environmental liability insurance illustrates an interaction between these two dimensions.

An interesting example of the role of insurance in dealing with ambiguity of both the probability and magnitude of losses is illustrated by a sales promotion campaign conducted by the Toro Company in 1983. Toro agreed to refund the entire price of its snow blowers to any customer who bought a machine before December 10, 1983 provided that the winter's snowfall amounted to less that 20% of the average in the location where the machine was bought. For customers, there was undoubtedly considerable ambiguity concerning what 20% of the average snowfall was in their city. However, the opportunity to receive a full refund if there was little snow in the coming winter helped sales skyrocket (Richards, 1983). In this case, Toro knew considerably more about the probability of different amounts of snow during the coming winter than did their customers; they also diversified their risk by selling machines across the country. However, their apparent concern with ambiguity regarding both the probability and magnitude of their potential losses led them to purchase insurance against a winterless winter from Good Weather International. Premiums were not disclosed.

A related example involved a promotional campaign in Belgium by one Europe's largest television manufacturers. [15] They offered to pay for any of their sets purchased during a six month period between the announcement of the 24 teams competing in the final rounds of soccer's 1986 World Cup and the start of the competition, provided Belgium was the overall winner. Although Belgium was a long-shot at odds of 25 to 1,[16] the television manufacturer was willing to pay a premium of approximately $30,000 to insure against a $300,000 loss should Belgium win the competition. In other words, the television manufacturer was willing to pay a premium that was more than 2 and 1/2 times greater than the expected loss that could have been estimated from the bookmakers' odds.[17]

By interviewing actuaries, and examining both actuarial procedures and comments written on questionnaires, the present study provides insight into the process of *how* actuaries deal with ambiguity on probability and losses in making decisions as opposed to just the outcomes of decisions (i.e., stated prices). As noted, this seems to be a two-stage process involving, first, a screening decision as to whether a proposed risk meets the desirable characteristics of the insurer's portfolio of risks, and then, if acceptable, the setting of the price. For the latter, there was convincing evidence of reliance on an anchor value equal to the expected value of the risk, followed by a subjectively determined adjustment to account for factors specific to the particular risk such as the presence of ambiguity and correlation between risks. Thus, the process is quite similar to the general strategy of decision making under ambiguity suggested by Einhorn and Hogarth (1985; 1986) although some details differ (i.e., the anchoring-and-adjustment process is applied to the price rather than just its probabilistic component).

From a methodological perspective, two aspects of the present study should be noted. First, to the best of our knowledge the study is unique in its attempt to gain data on the decision processes of *experts* dealing with ambiguity in their area of expertise (for details of processes in "everyday" inference, see Einhorn & Hogarth, 1985; Curley, Yates & Abrams, 1986). Second, an acute problem of conducting experimental research on risk and uncertainty lies in the difficulty of obtaining valid information concerning attitudes toward potential losses. One cannot, for example, allow experimental subjects to lose large sums of money. By studying hypothetical decisions of professionals used to dealing in losses, however, this paper makes a useful contribution to resolving this problem.

In addition to emphasizing the need to obtain more data on the details of the judgmental processes used in setting prices under ambiguity, our work suggests the need for a more general model of choice that could encompass the behavior observed in this study. Over and above the ability to explain the effects of ambiguity, we note at least two requirements of such a model. First, unless one is willing to live with the notion that people have different utility functions in ambiguous and non-ambiguous circumstances (cf. Smith, 1969), the model should permit nonadditive probabilities (for further discussion on this point, see Hogarth & Kunreuther, in press). Second, models should be able to capture effects due to the size of potential payoffs (or losses) which could result either from correlation between risks or, when risks are independent, the size of potential losses per se. From the viewpoint of the Einhorn-Hogarth ambiguity model, it is feasible to capture the above requirements by generalizing the model to situations involving non-ambiguous probabilities and, indeed, work has already progressed on this front (see Einhorn & Hogarth, 1986; Hogarth & Einhorn, 1988). The key notion here is that, even in the presence of precise, non-ambiguous probabilities, people still engage in a mental simulation process. This, in turn, implies that attitudes toward *uncertainty* including ambiguity can affect judgments of "probabilities" and thus prices.

Our findings also raise a broader set of questions as to the decision processes of individuals in organizational settings where their inputs are only part of the final choice process. The interesting study by MacCrimmon and Wehrung (1986) on managerial risk-taking indicates that managers want to collect additional information and delay decisions that involve uncertain risks. As further pointed out by March and Shapira (1987), managers take actions with the goal of controlling risks thereby suggesting that they want to avoid ambiguity if at all possible.

As discussed earlier, March and Shapira (1987) also note that, in making decisions, managers may focus attention on focal values such as target or even survival levels of performance. Moreover, these factors may be particularly important if the manager's performance is judged by others in the organization. In the case of pricing insurance, the decision process for new risks such as those used in our questionnaire involve multiple parties: the premiums estimated by actuaries are inputs into decisions made by underwriters as to whether or not to market a particular type of coverage and, if so, at what price. Hence the actuaries are aware that the premiums they recommend will be judged by other people. It is thus not surprising that they would raise their prices when probabilities are ambiguous particularly if potential losses are large and could impact target levels of performance such as the adequacy of reserves.

These findings suggest the need to gain a better understanding of the types of interactions between key individuals in firms (e.g., actuaries and underwriters) who have the responsibility for pricing and offering coverage against specific risks. What types of constraint impact on their decision processes? How are their actions judged by their superiors? Are there other attributes such as potential regret which influence their final choices? The importance of justification, uncertainty avoidance and other behavior which may not fit directly into the standard utility paradigm suggests a broad agenda for studying the impact of ambiguity and uncertainty by individuals in organizational settings.

Finally, the presence of ambiguity raises the broad question as to whether economic agents who follow heuristic rules of thumb can survive and prosper in a competitive environment. Recently, this and similar questions have aroused considerable interest in connection with stock market behavior where the existence of several anomalies seem to suggest that the market is not efficient.[18] The lack of availability of insurance in the past few years for a number of different risks raises a set of related questions regarding the performance of insurance markets. Ambiguous probabilities associated with potential losses may be an important determinant of this behavior. If insurers are uncertain as to the likelihood of specific events occurring and there is only limited opportunity to learn from experience due to the low probability nature of the risks, then they may be reluctant to insure except at a price considerably above expected value. By incorporating ambiguity more explicitly into models of choice we may be able to gain insight into the conditions under which thick and thin markets are likely to emerge and the resulting equilibrium values.

Appendix A

<u>Proof that the expected utility model implies that, for certain utility functions, premiums will be larger under ambiguity</u> [5]

Consider the situation where a firm with a nonlinear, risk-averse utility function is insuring m statistically independent risks each of which can result in a loss of L with probability p. Assume that in one case p is known unambiguously whereas in another an ambiguous estimate of p results from combining k estimates p_i ($i = 1,..., k$) provided by k experts. Further, impose the restriction that the unambiguous estimate $p = \sum_{i=1}^{k} w_i p_i$ where $\sum_{i=1}^{k} w_i = 1$ and $w_i \geq 0$ ($i = 1,...., k$).

Denote the random variable representing the total loss from all m risks in the non-ambiguous case by X(p). This is distributed according to a binomial distribution with mean of Lmp and variance of $L^2mp(1-p)$.

In the ambiguous case, the analog to X(p) is the random mixture of the k loss distributions $X(p_i)$, $i = 1,..., k$, each of which is a binomial random variable (multiplied by a constant L). Denote this random mixture by $\otimes_i X(p_i)$.

We first note that the means of the distributions of X(p) and $\otimes_i X(p_i)$ are the same since the absolute moments of mixture random variables are simply the corresponding mixture of absolute moments of their component random variables. However, if the variance of $\otimes_i X(p_i)$ is greater than that of X(p), this will imply that firms with nonlinear, risk-averse utility functions will wish to charge more for premiums in the presence of ambiguity (as operationalized here).

To show that $\mathrm{Var}[\otimes_i X(p_i)] \geq \mathrm{Var}[X(p)]$, note that

$$\begin{aligned}
\mathrm{Var}[\otimes_i X(p_i)] &= E\{[\otimes_i X(p_i)]^2\} - [E\{\otimes_i X(p_i)\}]^2 \\
&= \sum_{i=1}^{k} w_i E\{[X(p_i)]^2\} - [\sum_{i=1}^{k} w_i E\{X(p_i)\}]^2 \\
&= \sum_{i=1}^{k} w_i L^2[m(m-1)p_i^2 + mp_i] - [\sum_{i=1}^{k} w_i Lmp_i]^2 \qquad \text{(A.1.)}
\end{aligned}$$

Recalling that $Var[X(p)] = L^2mp(1-p)$, we can write,

$$Var[\otimes_i X(p_i)] - Var[X(p)]$$

$$= \sum_{i=1}^{k} w_i L^2 m(m-1)p_i^2 + L^2mp - [Lmp]^2 - L^2mp(1-p)$$

$$= [L^2m(m-1)]\{(\sum_{i=1}^{k} w_i p_i^2) - p^2\} \tag{A.2}$$

Noting that p^2 is a strictly convex function, it must be the case that $(\sum_{i=1}^{k} w_i p_i^2) > p^2$ except in the (trivial) case where all experts' opinions are identical (i.e., $p = p_i$ for all $i = 1,..., k$). Thus, since the expression in equation (A.2) is always nonnegative, it must be the case that firms with nonlinear, risk-averse utility functions will charge higher premiums in the presence of ambiguity.

Footnotes

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[1] In a stimulating article on recent developments in choice under uncertainty, Machina (1987) points out that until fifteen years ago expected utility theory was considered one of the success stories of economic analysis. Today, the theory is in a state of flux with economists and psychologists providing experimental evidence that it does not describe behavior (see, e.g., Schoemaker, 1982; Hogarth & Reder, 1987) and attempting to develop new theoretical models (for an overview, see Weber & Camerer, 1987).

[2] This issue has been clearly recognized in the financial press. See, for example, the article by Large (1984) in the *Wall Street Journal* (June 21, 1984). Similarly, the French magazine *L'Expansion* (June 6-12, 1986) reports great variations in the price of satellite insurance between 1983 and 1986 with premiums varying between 18% and 30% of the value of satellites.

[3] This definition of ambiguity would be considered as vague uncertainty by Budescu and Wallsten (1987) who distinguish between precise and vague uncertainties. They define a precise uncertainty as one that can be expressed as either a point probability estimate or a second order distribution over probability values. All other uncertainties are considered to be vague.

[4] We note, in particular, that pressures inside a firm can lead agents (e.g., actuaries) to adopt conservative attitudes toward risk. For example, actuaries may be held partially responsible for losses that the insurer may suffer and this could impact negatively on their jobs. Such behavior would be closely related to safety-first models of firm behavior (see e.g., Day, Aigner, & Smith, 1971) which have also been used to characterize insurance decision processes (see Stone, 1973).

[5] The essential intuition underlying the reasoning given in Appendix A is that the variance of a weighted average of probabilities is greater than the variance of the average itself. We are indebted to Paul Kleindorfer for formally proving the result in Appendix A.

[6] In what follows, we focus on *pure premiums*. This excludes considerations of factors such as charging administrative expenses, taking account of investment income from premiums received, and so on.

[7] Other approaches are to use coinsurance and deductibles so that the insured absorbs part of any given loss.

[8] Exceptions to this statement occur for large probabilities of losses and small probabilities of gains. For the former, whereas possible values above the anchor are weighted more heavily in imagination than those below, the range of possible values below the anchor is so much greater than that above that the net effect of the adjustment is negative. For small probabilities of gains it is the reverse. For fuller details, see Hogarth (in press).

[9] To standardize interpretation respondents were explicitly told that use of the words *pure premium* should be understood as meaning premiums *exclusive of all loss adjustment and underwriting expenses.*

[10] Copies of the questionnaires used in this paper can be obtained by writing to the authors.

[11] We tested for all possible two-way and three-way interactions involving probability level. Of these, only the two-way interaction between risk type (independent versus correlated) and probability level was significant. (See Table 2). However, we make no substantive interpretation of this particular result.

[12] This was for "the defective product scenario" described in Hogarth and Kunreuther (1985; in press).

[13] We would like to express our appreciation to Kenneth Frohlich, Robert DeLiberatto, Rich Ernst, Brian Moore (all of Reliance Corporation) and Jean Lemaire (Department of Insurance, Wharton School) for their participation in this process.

[14] For a detailed discussion of the conditions of insurability, see Berliner (1982).

[15] Personal discussion with Jean Lemaire.

[16] William Hill & Sons, Bookmakers, London, U.K., personal communication, May 1988.

[17] Interestingly enough, Belgium reached the semi-finals before losing.

[18] See DeLong et al. (1988) for a summary of these studies. These authors also show with a simple model that irrational traders with erroneous stochastic beliefs both affect prices and earn higher expected returns.

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Table 1

Predictions of different models

Predictions	Expected utility [a]	Actuarial procedures
1. Effect of ambiguity on independent risks	$r_2 > r_1$	$r_2 > r_1$
2. Effect of ambiguity on correlated risks	$r_3 = r_4$	$r_4 > r_3$
3. Effects of correlated vs. independent risks	$r_3 > r_1$ and $r_4 > r_2$	$r_3 > r_1$ and $r_4 > r_2$
4. Effect of increasing m, the size of potential loss		
a. Independent risks	?	$r_{ms} < r_{ml}$
b. Correlated risks	$r_{ms} < r_{ml}$	$r_{ms} < r_{ml}$

Legend: r_1 = premium for independent risks with non-ambiguous probabilities
r_2 = premium for independent risks with ambiguous probabilities
r_3 = premium for perfectly correlated risks with non-ambiguous probabilities
r_4 = premium for perfectly correlated risks with ambiguous probabilities
r_{ms} = premium for small number of risks
r_{ml} = premium for large number of risks
[a] Assuming a risk-averse utility function

Table 2

Mean (median) prices in the different experimental conditions

		Probabilities of breakdown .001	.010	.100
Expected value per unit loss:		$0.10	$1.00	$10.00
10,000 units		$	$	$
Independent:				
Non-ambiguous	(r_1)	0.23 (0.11)	1.18 (1.00)	11.05 (10.00)
Ambiguous	(r_2)	1.72 (0.40)	3.35 (1.50)	12.01 (12.00)
Correlated:				
Non-ambiguous	(r_3)	1.45 (0.70)	6.94 (2.25)	18.06 (14.00)
Ambiguous	(r_4)	2.19 (1.00)	6.67 (5.00)	20.24 (20.00)
100,000 units				
Independent:				
Non-ambiguous	(r_1)	0.97 (0.12)	1.11 (1.05)	10.26 (10.00)
Ambiguous	(r_2)	1.69 (0.50)	2.99 (2.00)	16.56 (12.00)
Correlated:				
Non-ambiguous	(r_3)	4.15 (0.10)	5.24 (1.24)	16.46 (12.25)
Ambiguous	(r_4)	3.11 (1.00)	13.00 (12.85)	26.18 (25.00)

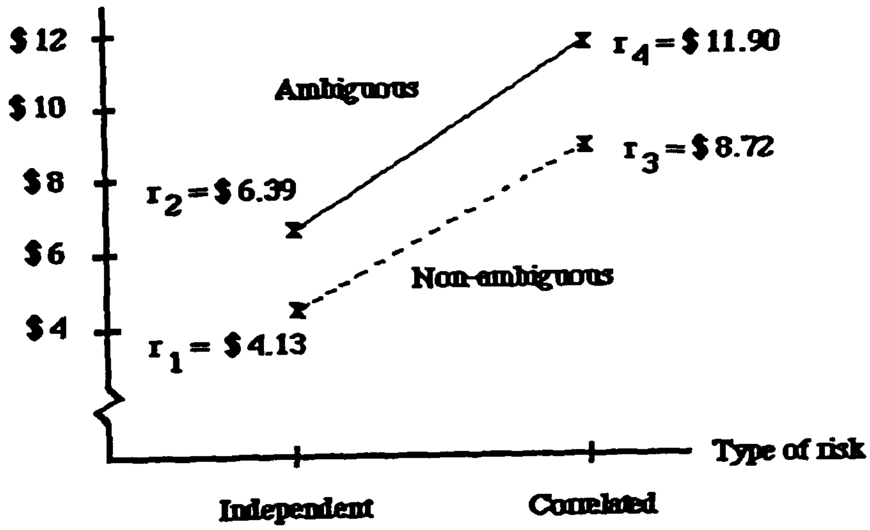
Note: Variables and interactions significant by ANOVA

	df	F value	p level
Number of units	1	2.80	0.095
Ambiguity	1	10.73	0.001
Risk type (independent vs. correlated)	1	37.11	< 0.001
Probability level	2	115.35	< 0.001
Number of units x ambiguity	1	3.30	0.070
Risk type x probability level	1	4.49	0.012

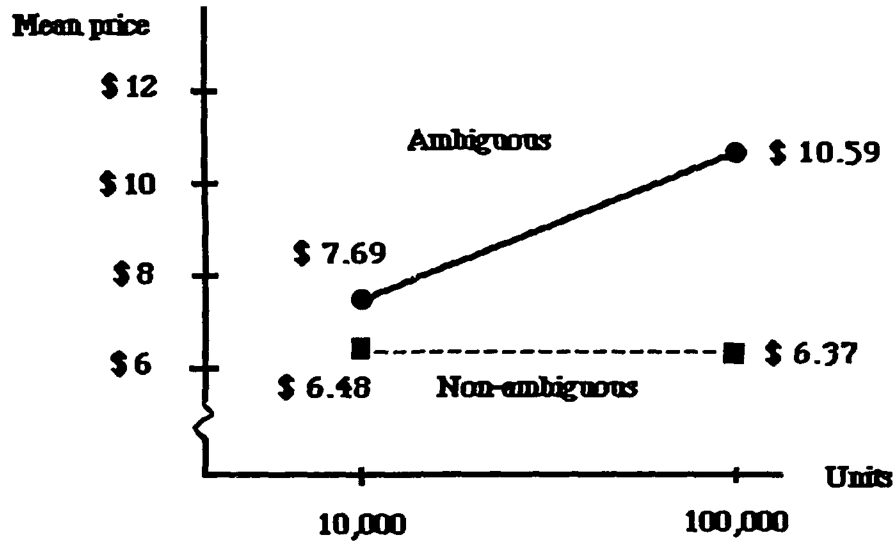
Figure Captions

Figure 1: Mean prices in ambiguous and non-ambiguous conditions by type of risk (i.e., independent versus correlated).

Figure 2: Interaction of ambiguity and size of stake (i.e., number of units).



$ 12
$ 10
$ 8
$ 6
$ 4
Ambiguous
Non-ambiguous
r2 = $ 6.39
r1 = $ 4.13
r4 = $ 11.90
r3 = $ 8.72
Type of risk
Independent
Correlated


Mean price
$ 12
$ 10
$ 8
$ 6
Ambiguous
$ 10.59
$ 7.69
$ 6.37
$ 6.48
Non-ambiguous
Units
10,000
100,000

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